Resistive Transition Equation of Mixed State: A Microscopic Analysis

Daole Yin*, Zhi Qi, Hengyi Xu and Furen Wang

Department of Physics, Peking University, Beijing 100871, P. R. China

Lan Yin

Department of Physics, University of Washington, Box 351560 Seattle, WA98195-1560

Abstract

We present a microscopic derivation of the resistive transition equation for mixed state of superconductors. This form fits the experimental data of MgB_2 with parameters in agreement with the prediction of BCS superconductivity. It also fits the experimental data of high quality untwinned YBCO single crystal but with parameters somewhat different from the BCS prediction. A discussion in connection with the problem of cuprate superconductivity is given.

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I. INTRODUCTION

The discovery of high-temperature superconductors (HTS) revived the interest in the mixed state physics. One of the most interesting issues is the resistive transition behavior of a superconductor. The heuristic pioneering work of Tihkham [1] showed that by using a proper activation energy for flux flow the experimentally measured early field-depending broadening of HTS [2] can be accounted for nicely by the model of thermally activated flux flow model (TAFF) [3]. However, subsequent measurements with higher sensitivity showed that the fit was quantitative only down to one tenth of the normal resistance $R \sim 0.1R_n$. A number of authors suggested that the much more rapid drop of resistance than the exponential predicted by the TAFF model might be due to a freezing / melting transition between a vortex liquid and a vortex solid of some sort [4]. Koch et al. [5] and subsequently Gammel et al. [6] carefully tested experimentally the predictions of the 'vortex-glass model' proposed by M. P. A. Fisher [7] by measuring the $I \sim V$ curves and resistance of YBCO epitaxial thin films and single crystals with high sensitivity. They found that for each filed B the isothermal $I \sim V$ curve shows a power-law behavior at a uniquely defined temperature $T_g(B)$ as

$$V \propto I^n. \tag{1a}$$

At $T > T_g(B)$, the curvature on a $\ln V$ vs $\ln I$ plot is positive corresponding to the form of TAFF models prediction

$$V \propto \sinh\left(\frac{I}{I_0}\right). \tag{1b}$$

In contrast, at $T < T_g(B)$ one finds a negative curvature with the characteristic predicted by vortex-glass model [7]

$$V \propto \exp\left[-\left(\frac{J_T}{J}\right)^{\mu}\right]. \tag{1c}$$

Since $T_g(B)$ is defined as the dividing point between temperatures for which the linear resistance (i.e., R in the limit of $I \to 0$) is zero, as indicated by Eq. (1c), and that in

which it is not zero, as Eq. (1b), this transition temperature $T_g(B)$ is operationally very much the same as the melting temperature of vortex solid $T_m(B)$ measured in (nearly) ideal crystals. Kwok et al. observed a sharp 'kink' or 'knee' in the magnetic-field-broadened resistive transition at $R/R_n \lesssim 0.12$ in a high-quality untwinned YBCO single crystal. This behavior obeys the angular dependence expected from the Lindemann criterion of vortex lattice melting [8]. The resistive transition broadening as well as the 'irreversibility line' were also observed in the recently discovered MgB_2 with $T_c \sim 40K$ [9], though it shows typical behavior of a phonon-mediated BCS superconductor through the B isotope effect [10].

Recently, it is found that the resistive transition of both HTS and MgB_2 can be well described with a equation of the normalized form

$$\frac{R}{R_n} = \exp\left[-\sum_{i=1}^{2}\gamma_i\left(1+y_i-x_i\right)^{p_i}\theta\left(T_i-T\right)\right], \tag{2}$$

with x_i, y_i, γ_i and T_i the normalized current, voltage, symmetry-breaking factor and critical temperature respectively defined as

$$\begin{aligned}
x_1 &\equiv \frac{I}{I_d(T,\,B)}, & x_2 &\equiv \frac{I}{I_{c0}(T,\,B)} \\
y_1 &\equiv \frac{R}{R_n}\left[\frac{I}{I_d(T,\,B)}\right], & y_2 &\equiv \frac{R}{R_f}\left[\frac{I}{I_{c0}(T,\,B)}\right] \\
\gamma_1 &\equiv \ln\frac{R_n(T,\,B)}{R_f(T,\,B)}, & \gamma_2 &\equiv \frac{U_c(T,\,B)}{kT} \\
T_1 &\equiv T_c(B), & T_2 &\equiv T_m(B)
\end{aligned} \tag{3}$$

where I_d is the depairing current and I_{c0} is the critical current of vortex solid for overcoming the activation energy barrier $U_c(T,\,B)$. $\theta(x)$ is the Heaviside function and p_i are exponents [11].

In present work we try to show the connection of the resistive transition equation (2) with the basic Ginzburg-Landau (G-L) theory [12]which can be derived from the microscopic theory of inhomogeneous superconductor as shown by Gorkov [13].

In sections II and III we shall derive the two terms in the right hand side braket of Eq. (2), i.e., $i = 1$ and 2, in connection with the normal-superconducting state (N-S) transition and

flux pinning respectively. The derived equation will be compared with the experimental data of MgB_2 and YBCO single crystal in section IV. A discussion about possible implications is given in section V.

II. TRANSITION NEAR $B_{c2}(T)$

The basic frame for describing the physics of superconducting mixed state is the Ginzburg-Landau (GL) free energy density f in the form [12]

$$f = f_{n0} + \alpha \left|\Psi\right|^2 + \frac{\beta}{2}\left|\Psi\right|^4 + \frac{1}{2m^*}\left|\left(\frac{\hbar}{i}\nabla - \frac{e^*}{c}\mathbf{A}\right)\Psi\right|^2 + \frac{h^2}{8\pi}, \tag{4}$$

where f_{n0} is the free energy density of normal state, $\Psi(r)$ is the complex order parameter, $\mathbf{A}$ is the vector potential. Gorkov found that the GL theory based on Eq. (4) is derivable as a rigorous limiting case of the BCS microscopic theory with $\Psi(r)$ proportional to the local value of the gap parameter $\Delta(r)$ and the effective charge e^* in Eq. (4) equal to $2e$ [13]. Within the framework of GL theory Bardeen and Stephen studied the flux flow in a pinning free mixed state and obtained the flow resistance ρ_f as [14]

$$\frac{\rho_f}{\rho_n} = \frac{2\pi a^2 B}{\Phi_0} = \left(\frac{a}{\xi}\right)^2 \frac{B}{B_{c2}} \approx \frac{B}{B_{c2}}, \tag{5}$$

where a is the radius of normal core in the local model, nearly equal to the G-L coherence length ξ which can be expressed in terms of Gorkov's derivation with BCS superconductivity as [13]

$$\xi^2(T) = \frac{\hbar^2}{2m^*\left|\alpha(T)\right|} = \frac{7\hbar^2\zeta(3)\,\epsilon_F\beta_c^2}{12\pi^2 m^*}\left(1 - \frac{T}{T_c}\right)^{-1}, \tag{6}$$

with $\zeta(3)$ the Rieman zeta function and $\beta_c \equiv 1/k_B T_c$.

For phonon mediated BCS superconductors the energy gap $\Delta(T)$ near T_c is shown as

$$\left(\frac{\Delta(T)}{\Delta_0}\right)^2 = 3.1\left(1 - \frac{T}{T_c}\right), \tag{7}$$

with $\Delta_0 \equiv \Delta(T=0) \equiv 1.76 k_B T_c$ [12,15]. Further study on the dependence of Δ on field and Cooper pair velocity v_s finds [15]

$$\begin{aligned} \left[\beta_c\Delta(T, v_s)\right]^2 &= \frac{8\pi^2}{7\zeta(3)}\left(1 - \frac{T}{T_c}\right) - \frac{2}{3}\left(\beta_c\hbar k_F\right)^2 v_s^2 \\ &= \left[\beta_c\Delta(T, 0)\right]^2 - \frac{2}{3}\left(\beta_c\hbar k_F\right)^2 v_s^2. \end{aligned} \tag{8}$$

Inserting (6), (7), (8) into (5) and considering the identity

$$\ln\left(1-x\right)=-\sum\frac{x^{n}}{n},\qquad n=1,2,...,\text{ with}-1<x\leqslant 1,\tag{9}$$

We find

$$\begin{aligned}\rho_{f}\left(T,B,v_{s}\right)&=\rho_{n}\left(T,B\right)e^{-\gamma_{1}}\exp\left[\ln\frac{\Delta^{2}\left(T,B,0\right)}{\Delta^{2}\left(T,B,v_{s}\right)}\right]\\&\approx\rho_{n}\left(T,B\right)\exp\left\{-\gamma_{1}\left[1-\frac{2\left(\hbar k_{F}\right)^{2}v_{s}^{2}}{3\gamma_{0}\Delta^{2}\left(T,B,0\right)}\right]\right\}.\end{aligned}\tag{10}$$

Using the relation $J_s = en_s v_s$ with the density of superconducting electrons $n_s \approx 2n\left(1-T/T_c\right) \approx 2n\Delta^2\left(T\right)/\pi\Delta_0^2$, one gets Eq. (10) in the form

$$\rho_{f}\left(T,B,J\right)=\rho_{n}\left(T,B\right)\exp\left[-\gamma_{1}\left(1-\frac{J_{s}^{2}}{J_{d}^{2}}\right)\right],\tag{11}$$

with

$$J_{d}^{2}\approx\frac{2\gamma_{0}e^{2}n^{2}\Delta^{6}\left(T,B,0\right)}{\left(\hbar k_{F}\right)^{2}\pi\Delta_{0}^{4}}\propto\gamma_{1}T_{c}^{2}\left(1-\frac{T}{T_{c}}\right)^{3}.\tag{12}$$

Eq. (11) is the same form of Eq. (2) in the pinning-free ($\gamma_2 = 0$) case with the evaluation of average supercurrent J_s as

$$J_{s}=J-\frac{E\left(J\right)}{\rho_{n}\left(T,B\right)}.\tag{13}$$

III. RESISTANCE DUE TO FLUX CREEP

Working in the London limit, Nelson showed that according to the GL free energy Eq. (4) a system of N flux lines with a field H along the z direction in a sample length L can be described with the free energy represented by the trajectories $\left\{\vec{r}_j\left(z\right)\right\}$ of these flux lines [16–18]. Considering further the pinning potential $V_P\left(\vec{r}\right)$ arising from inhomogeneities and defects in sample [7,19,20], the free energy of such a sample with N flux lines can be expressed as

$$F=\frac{1}{2}\varepsilon_{l}\sum_{j=1}^{N}\int_{0}^{L}\left|\frac{d\,\vec{r}_{j}\left(z\right)}{dz}\right|^{2}dz+\frac{1}{2}\sum_{i\neq j}\int_{0}^{L}V\left(r_{ij}\right)dz+\sum_{j=1}^{N}\int_{0}^{L}V_{P}\left[\vec{r}_{j}\left(z\right)\right]dz,\tag{14}$$

Here $V(r_{ij}) = V\left(\left|\vec{r_i} - \vec{r_j}\right|\right) = 2\varepsilon_0 K_0 (r_{ij}/\lambda_{ab})$ is the interaction potential between lines with in-plane London penetration depth λ_{ab} and $K_0(x)$ is the modified Bessel function $K_0(x) \approx (\pi/2x)^{1/2} e^{-x}$. ε_l is the linear tension of flux line and $\varepsilon_0 \approx (\Phi_0/4\pi\lambda_{ab})^2$ is the energy scale for the interaction.

Thermally activated flux motion is considered as the sequence of thermally activated jumps of the vortex segments or vortex bundles between the metastable states generated by disorder. Every elementary jump is viewed as the nuclearation of the vortex loop, and the mean velocity of the vortex system is determined by the nuclearation rate [7,19,20]

$$v \propto \exp(\delta F/kT). \tag{15}$$

Here δF is the free energy for the formation of the critical size loop or nucleus which can be found by means of the standard variational procedure from the free energy functional due to the in-plane displacement $\vec{u}(z)$ of the moving vortex during loop formation

$$F_{loop}\left[\vec{u}\right] = \int dz \left[\frac{1}{2}\varepsilon_l \left|\frac{d\,\vec{u}(z)}{dz}\right|^2 + V_P\left[\vec{u}(z)\right] - (f_L + f_\eta)\cdot\vec{u}\right], \tag{16}$$

where $f_L = \vec{J} \times \vec{e_z}/c$ is the Lorentz force due to applied current J and f_η is the viscous drag force on vortex, $f_\eta = -\eta v_{vortex}$, with $v_{vortex} = d\,\vec{u}/dt$ and viscous drag coefficient $\eta \approx (\Phi_0 B_{c2})/(\rho_n c^2)$ as estimated by Bardeen and Stephen [14].

Equation (16) is similar to the basic equations of collective pinning model and vortex-glass as well as Bose-glass models [21] for considering the vortex dynamics at low temperatures. The only difference is the omission of f_η in the latter, which is quite resonable since the velocity of vortex v_{vortex} usually is very small in the case of low temperature flux creep, where one finds $f_\eta \ll f_L$. However, we have to consider the term f_η in our equation (16) not only because the fact that viscous drag force does ever accompany the vortex displacement as shown experimentally by Kunchur et al. [22] but also for the need in our task to describe the current-voltage characteristic of vortex system more accurately for the case across the irreversibility line where f_L and f_η are comparable in magnitude, so now we have

$$F_{loop}\left[\vec{u}\right]=\int dz\left[\frac{1}{2}\varepsilon_l\left|\frac{d\,\vec{u}\,(z)}{dz}\right|^2+V_P\left[\vec{u}\,(z)\right]-f_s\cdot\vec{u}\right], \tag{16'}$$

with

$$f_s=f_L+f_\eta=\frac{J_P\Phi_0}{c}\times\vec{e_z}\qquad\text{and}\qquad J_P=J-\frac{E}{\rho_f}, \tag{17}$$

where we used the relation $E=\vec{v}\times\vec{B}$ and Eq. (5).

The free energy functional $F_{loop}\left[\vec{u}\right]$ contains two parts. The last term in the right hand side of Eq. (16) represents the net energy gained from the applied current which is proportional to J_p in Eq. (17) and the rest are corresponding to the increase of the elastic energy during the loop formation. As argued by Fisher et al. [7], in a bulk superconductor vortex lines are extended one-dimensional (1D) objects and the response of vortices to an applied current can be described by vortex loop formation of area $S=L_\perp\cdot L_z\sim L_\perp^\kappa$ and elastic energy increase $\sim U_c\cdot L_\perp^\theta$, where $L_\perp$ is the transverse displacement of vortex-line segment of length L_z, with scaling relation $L_\perp\sim L_z^\zeta$.

Consequently, the free energy of loop formation can be estimated in the dependence of $L_\perp$ as

$$F_{loop}\left(L_\perp\right)\approx U_cL_\perp^\theta-J_p\frac{\Phi_0}{c}L_\perp^\kappa. \tag{18}$$

For a given value of J_p, $F_{loop}\left(L_\perp\right)$ first increases and then decreases with increasing $L_\perp$ (since $\kappa>\theta$ and $J_p<J_c$). The barrier energy is the maximum value of loop formation free energy

$$\delta F=U\left(J_p\right)=F_{loop}\left[L_\perp^*\left(J_p\right)\right], \tag{19}$$

with definition of $L_\perp^*\left(J_p\right)$

$$\left.\frac{\partial F_{loop}\left(L_\perp\right)}{\partial L_\perp}\right|_{L_\perp=L^*}=0,\qquad L_\perp^*\left(J_p\right)=\left(\frac{c\theta U_c}{\Phi_0\kappa J_p}\right)^{1/(\kappa-\theta)}. \tag{20}$$

Combining Eqs. (18), (19) and (20) one finds

$$U\left(J_p\right)\approx U_c\left(\frac{J_c}{J_p}\right)^\mu, \tag{21}$$

where

$$J_c \sim \frac{\theta U_c c}{\kappa \Phi_0} \qquad \text{and} \qquad \mu \sim \frac{\theta}{\kappa - \theta} = \frac{2\zeta - 1}{2 - \zeta}. \tag{22}$$

The current dependence (21) of the barrier U to current-driven flux motion implies a current-voltage characteristic of the form

$$E(J) = \rho_f J \exp\left[-\frac{U_c}{kT} \left(\frac{J_c}{J_p} \right)^{\mu} \right]. \tag{23}$$

This form in one hand turns to the pinning-free flux flow resistive regime as $J, J_p \gg J_c$ and clearly goes to zero in the other hand as $J_p \sim J \to 0$, with no linear resistance term.

In real samples, the critical size of loop formation $L_{\perp}^{*}(J_p)$, and thus $U(J_p)$ may always manifest a nonzero resistance at sufficiently small measuring current with high enough accuracy of measurements. Considering this real size effect one finds a general normalized form of the current-voltage characteristic in the form (see Appendix)

$$y = x \exp\left[-\gamma (1 + y - x)^{p} \right], \tag{24}$$

with

$$\gamma = \frac{U_c}{kT} \left(\frac{J_c}{J_L} \right), \qquad x = \frac{J}{J_L}, \qquad y = \frac{E(J)}{\rho_f J_L}, \qquad p = \mu. \tag{25}$$

Where J_L is the transport current density corresponding to the case where the critical size of loop formation is equal to the sample size L as

$$\left(\frac{c\theta U_c}{\Phi_0 \kappa} \right) \left[J_L - \frac{E(J_L)}{\rho_f} \right]^{-1} = J_c \left[J_L - \frac{E(J_L)}{\rho_f} \right]^{-1} \approx \frac{J_c}{J_L} = L^{\kappa - \theta}. \tag{26}$$

Together with the former equation (11) in section II, equation (24) is just the normalized form of the resistive transition equation (2) in the case of temperatures below the melting point of vortex solid $T < T_2 = T_m(B)$ with $p_2 = \mu$ and $J_{c0}(T, B) = J_L(T, B)$.

IV. COMPARISON WITH EXPERIMENTS

In this section we compare the above derived resistive transition equation of mixed state in the form of Eq. (2) with the experimental data of several typical superconductors with high T_c.

A. MgB_2

The recently discovered superconducting MgB_2 has a high critical temperature $T_c \sim 40K$ comparable to the cuprate $LaSrCuO$. Meanwhile, it shows isotope effect of phonon-mediated superconductivity. Finnemore et al. measured the transport and magnetic properties of sintered pellet of MgB_2 and find the Ginzburg-Landau parameter $\kappa \sim 26$ [9]. In Fig. 1 we show the comparison of the resistive transition equation (2) with their experimental data of the temperature dependent resistance of MgB_2 from 300K to 1.9K in different applied fields.

B. $YBa_2Cu_3O_{7-\delta}$ (YBCO)

It is widely believed that the mechanism of superconductivity in high-T_c cuprates may be essentially different, from the familiar s-wave type pairing on which conventional BCS theory is based [22]. Kwok et al. studied the width and shape of the resistive transition of untwinned and twinned single crystals of YBCO in fields up to $8T$ [8] .This is one of the first evidences which came from transport measurements for vortex melting in YBCO. The samples are of high quality and near optimum doping confirmed by their zero-field resistive transition of $R_{zero} > 92.0K$ and $\Delta T_c\,(10\% - 90\%) < 0.2K$. A "knee" in $R\,(T)$ curve is clearly seen. We compare our resistive equation (2) with their experimental data of untwinned YBCO crystal in Fig. 2.

Agreement is fair in both cases.

V. DISCUSSION

Though the resistive transition equation (2) fits the experimental data of both MgB_2 and YBCO crystal, there are still essential differences in the parameters entered into Eq. (2) for these two kinds of materials. In Fig. 1 we see the parameters used to fit the experimental data of MgB_2 by Finnemore et al. [8] follow the relations $I_d \propto \gamma_1^{0.5}\left[T_c\,(B) - T\right]^{1.5}$ and $\gamma_1 \equiv \ln\left[R_n\,(T,B)\,/R_f\,(T,B,J \to 0)\right] = \ln\left[B_{c2}\,(T)\,/B\right]$ in agreement with Eq. (17) and the flux-flow resistance equation derived by Bardeen and Stephen [4] for BCS superconductivity. In contrast, the parameters to fit the experimental data of YBCO crystal by Kwok et al. [8]show the relations $I_d \propto \gamma_1^{0.5}\left[1 - T/T_c\,(B)\right]^{2.3}$ and $\gamma_1 \equiv \ln\left[R_n\,(T,B)\,/R_f\,(T,B,J \to 0)\right] =$

$\ln \left[B_{c2}\left(T\right)/B\right]^{m}$, with $m = 0.84B^{-0.59} - 0.07$ which are somewhat different from Eq. (17) and the results of Bardeen and Stephen [4] in the exponents. At present a correct microscopic theory for the high-T_c cuprates is still a challenging problem in condensed matter physics. While it seems that the established superconductivity of cuperates is of $d_{x^2-y^2}$ symmetry, the next question is whether it can be described by a BCS- like theory suitably modified to in clude a d-wave gap or it is of some non-BCS origins [23]. On the one hand, there are recent reports on the evidences for strong electron-phonon coupling in high-T_c cuprates by the analysis of photoelectron spectra [24] and the unconventional isotope effect [25] as well as the identification of the bulk pairing symmetry in high-T_c hole-doped cuprates as extended s-wave symmetry with eight-line nodes and as anisotropic s-wave in electron-doped cuprates [26] which seem favouring the BCS-like machanism. On the other hand, the well established unusual properties of both the normal and superconducting states of cuprates seem related to the fact that the cuprates are doped Mott insulators and stimulate some ideas based on new exitations such as spin-charge separation [27,28], stripes [29], new symmetry relating superconductivity and magnetism [30] or quantum critical points [31] which suggest a non-BCS state. The interplay between theory and experiment promises to be mutually benificial, in the best traditions of physics research. In this paper we have compared the derived equation (2) only with the data of optimally doped YBCO single crystal [8]. However, the unusual properties of cuprates appear even more striking in the underdoped region. A thorough study on the resistive transition of underdoped cuprates may provide new important insights into the nature of cuprates.

VI. SUMMARY

Starting from the Ginzburg-Landau functional we derived a resistive transition equation for the mixed state of superconductors. This equation in its general form agrees with the experimental data of superconducting MgB_2 pellet and optially doped untwinned YBCO single crystal but the parameters to fit the data of cuprate are somewhat different from the prediction based on BCS theory.

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APPENDIX

In this appendix we provide the derivation of Eq. (24). Substituting Eq. (17) into the bracket on the right-hand side of Eq. (23) and taking its logarithm we get

$$J - J_f = \left(\frac{U_c}{kT}\right)^{1/\mu} J_c \left[\ln\left(\frac{J}{J_f}\right)\right]^{-1/\mu}, \tag{A.1}$$

with $J_f \equiv E(J)/\rho_f$. Since the critical size of loop formation $L_\perp^*$ is limited by the sample size L as $L_\perp^* \leqslant L$, one finds always

$$U(J) \leqslant U(J_L), \tag{A.2}$$

with the definition

$$\left(\frac{c\theta U_c}{\Phi_0 \kappa}\right)\left[J_L - \frac{E(J_L)}{\rho_f}\right]^{-1} \equiv L^{\kappa-\theta}. \tag{A.3}$$

Here we have used the relations in Eq. (17) and Eq. (20). Thus, Eq. (A.1) can be expressed in the form

$$J - J_f = \left(\frac{U_c}{kT}\right)^{1/\mu} J_c (1+h)^{-1} \left[\ln\left(\frac{J_L}{J_{Lf}}\right)\right]^{-1/\mu}, \tag{A.4}$$

where $J_{Lf} \equiv E(J_L)/\rho_f$, which is much smaller than J_L and

$$h \equiv \frac{-\left[\ln(J_L/J_{Lf})^{1/\mu} - \ln(J/J_f)^{1/\mu}\right]}{\ln(J_L/J_{Lf})^{1/\mu}}, \qquad |h| < 1. \tag{A.5}$$

Using the approximation $(1+h)^{-1} \approx 1 - h$ for $|h| < 1$, finally we find Eq. (A.1) in the form

$$x - y = 1 - \ln\left(\frac{x}{y}\right)^{1/p} \gamma^{-1/p}, \tag{A.6}$$

which is exactly the current-voltage characteristic Eq. (24) with the definitions

$$\begin{aligned}
\gamma &\equiv \ln\frac{J_L}{J_{Lf}} = \left(\frac{U_c}{kT}\right)\left(\frac{J_c}{J_L - J_{Lf}}\right) \approx \left(\frac{U_c}{kT}\right)\left(\frac{J_c}{J_L}\right)^p, \\
x &\equiv \left(\frac{U_c}{kT}\right)^{-1/p}\left(\ln\frac{J_L}{J_{Lf}}\right)^{1/p}\left(\frac{J}{J_c}\right) = \frac{1}{2}\frac{J}{J_L - J_{Lf}} \approx \frac{J}{J_L}, \\
y &\equiv \left(\frac{U_c}{kT}\right)^{-1/p}\left(\ln\frac{J_L}{J_{Lf}}\right)^{1/p}\left(\frac{E(J)}{J_0\rho_f}\right) = \frac{1}{2}\frac{E(J)}{(J_L - J_{Lf})\rho_f} \approx \frac{E(J)}{J_L\rho_f}.
\end{aligned} \tag{A.7}$$

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* Email address: ydl@ibm320h.phy.pku.edu.cn

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FIGURES

FIG. 1. Comparison of Eq.(19) with the experimental resistance data of MgB_2 samples measured by Finnemore et al. [8]. (a) A full view in the applied field of $9T$. (b)Resistive transitions in different applied fields. □ ◯ △▽∗ ◁▷ +★× denote experimental data and lines denote the theoretical curves of Eq.(19) with corresponding applied fields. The parameters in Eq.(19): 1 $U(T,B) \propto [T_m(B)-T]^{0.8}(B+5.46)^{-4.3}$. 2 $I_{c0} \propto T_m(B)-T$; $I_d \propto [T_c(B)-T]^{1.5}\gamma_0^{0.5}$. 3 $T_m(B) = T_c(0)\left[1-(B/21.7)^{0.84}\right]$, $T_c(B) = T_c(0)(1-B/22.4)$, $T_c(0) = 40.2K$. 4 $\gamma_0 \equiv \ln[R_n(T,B)/R_f(T,B,J\to 0)] = \ln[B_{c2}(T)/B]$, where $B_{c2}(T) = 0.6[T_c(0)-T]^{0.98}$.

FIG. 2. Comparison of Eq.(19) with the experimental resistive transition data of untwinned YBCO crystal measured in different applied fields for $H_{//c}$ by Kwok et al. [8]. □ ◯ △▽∗ ◁▷ +★× denote experimental data and lines denote the theoretical curves of Eq.(19) with corresponding applied fields. The parameters in Eq.(19): 1 $U(T,B) \propto [T_m(B)-T]^3 B^{-4.22}$. 2 $I_{c0} \propto T_m(B)-T$; $I_d \propto T_c(B)[1-T/T_c(B)]^{2.3}\gamma_0^{0.5}$. 3 $T_m(B) = T_c(0)\left[1-(B/1200)^{0.36}\right]$, $T_c(B) = T_c(0)\left[1-\left(B/3.6\times 10^6\right)^{0.3}\right]$, $T_c(0) = 97K$. 4 $\gamma_0 \equiv \ln[R_n(T,B)/R_f(T,B,J\to 0)] = \ln[B_{c2}(T)/B]^m$, where $m = 0.84B^{-0.59} - 0.07$ and $B_{c2}(T) = [T_c(0)-T]^{3.3}$.